Emergent Cosmology From Matrix Theory

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Standard Big Bang Cosmology is very successful at describing our current universe.

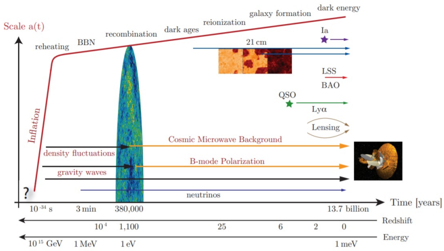


Figure: Brief overview of the thermal history of the universe

Some notable successes:

- Distant galaxies are red shifted.
- The CMB exists and we can extract a wealth of information from it.
- Abundance of light elements matches predictions from BBN.

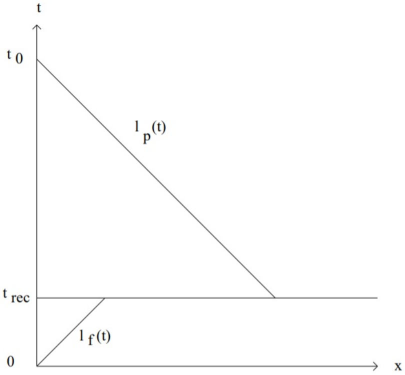
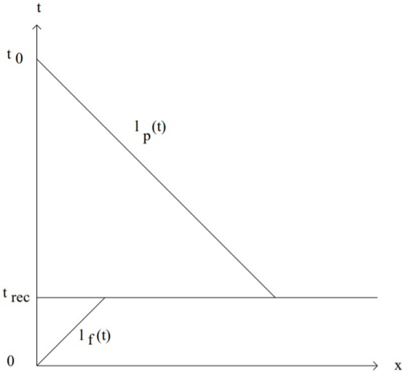


Figure: Sketch illustrating the horizon problem of SBB cosmology

Horizon Problem

Causality alone cannot explain the observed homogeneity our observed universe.



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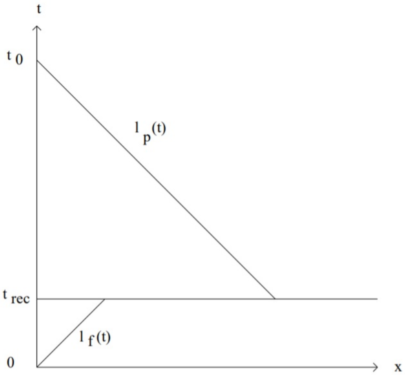
Flatness Problem

$$|\Omega - 1| \sim T^{-2}$$

$\Omega \approx 1$ requires extreme initial fine tuning!



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Singularity Problem

$$\rho \sim a^{-4}(t) \quad \text{(Radiation era)}$$

Energy density blows up when $a(t) \to 0$.

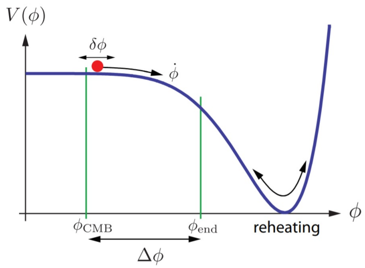
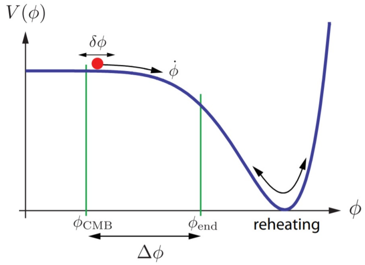


Figure: Example of inflaton potential. The inflaton slowly rolls down the potential until the conditions for inflation are broken.

A long enough period of inflation can solve the problems of SBB cosmology.



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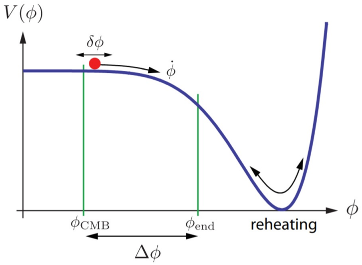

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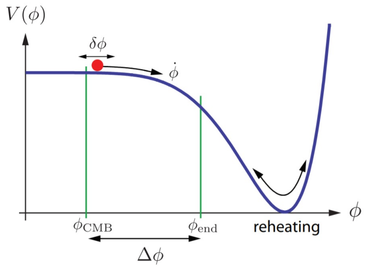
de Sitter conjecture: The scalar potential V is constrained by

$$|\nabla V| \geq cV \quad , \quad \min(\nabla_i \nabla_j V) \leq -c' V \, .$$



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TCC conjecture: The duration t of inflation is constrained by

$$t \leq H^{-1} \ln(H^{-1}) \, .$$

Singularity problem in string theory

Inflation does not solve the singularity problem. Moreover, the singularity problem is hard to deal with in perturbative descriptions of string theory.

$$S_{st} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} e^{-2\phi} \left[R + \frac{\alpha'}{8} R_{GB}^2 + \ldots \right] + S_{matter}$$

For cosmological solutions, the perturbative description holds when

$$R \gg \frac{\alpha'}{8} R_{GB}^2 \quad \Longrightarrow \quad t \gg \sqrt{\alpha'}$$

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Possible Solution

Look for new early universe cosmology scenarios based on non-perturbative approaches to string theory (e.g. Matrix descriptions of string theory).

Introduction to the IKKT matrix model

The IKKT model is a non-perturbative formulation of type IIB string theory in ten dimensions. The action of the IKKT model is given by

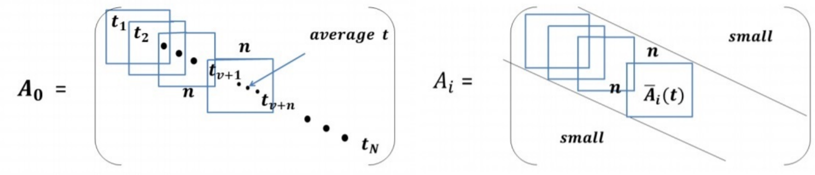
$$S_{IKKT} = -\mathrm{Tr}\left(\frac{1}{4g^2}[A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2g^2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right)$$

where A_μ ($\mu = 0, \dots, 9$) and ψ are $N \times N$ Hermitian matrices.

Here, space-time is described by the matrix elements A_0 and A_i, which hold information about time and space respectively.

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, , Nucl. Phys. B **498**, 467-491 (1997)]

To find dynamical solutions, we must minimize the IKKT action (including fermions) using Monte-Carlo methods. When A^0 is chosen to be diagonal, A^i has a band diagonal structure (see below).



Time parameter

$$t \equiv \frac{1}{n} \sum_{a=1}^{n} t_{\nu+a}$$

Time evolving matrix element

$$\bar{A}_i^{ab}(t) \equiv \langle t_{\nu+a} | A_i | t_{\nu+b} \rangle$$

[S. W. Kim, J. Nishimura and A. Tsuchiya, Phys. Rev. Lett. **108**, 011601 (2012)]

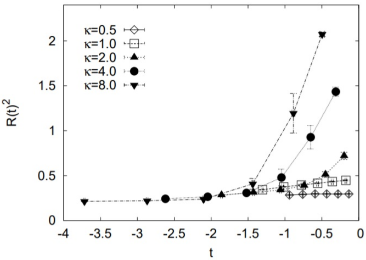


Figure: The extent of space $R(t)^2$ becomes large at a critical time t_c.

Extent of space parameter

$$R(t)^2 \equiv \frac{1}{n}\mathrm{Tr}\bar{A}_i(t)^2$$

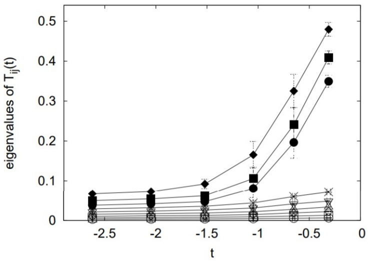


Figure: 3 out of 9 eigenvalues of T_{ij} become large at a critical time t_c.

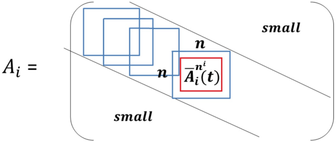
Moment of inertia tensor

$$T_{ij}(t) \equiv \frac{1}{n}\mathrm{Tr}\{\bar{A}_i(t)\bar{A}_j(t)\}$$

Emergent Space

Define a $n_i \times n_i$ matrix $\bar{A}_i^{n_i}$, which holds information about a comoving interval $[-n_i, n_i]$.

Space defined in this way has a few interesting properties.



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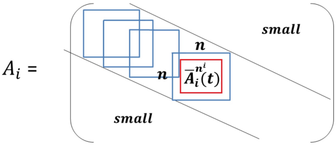

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The maximal comoving distance n_c scales as

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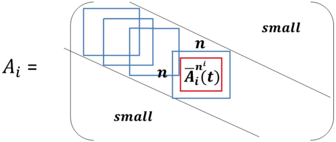

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Space is continuous

The interval δx between comoving coordinates scales as

$$\delta x \sim N^{-1/2} .$$

Space becomes continuous as $N \to \infty$.

To derive the metric, we first define the physical length of a curve in the interval $[-n_i, n_i]$ as

$$l_i^2(t, n_i) \equiv \langle \mathrm{Tr}(\bar{A}_i^{n_i}(t))^2 \rangle \,.$$

Then, we note that g_{ii} can be computed from the physical length along i in the following way

$$l_i(t, x) = \int_{y=-x}^{y=x} \sqrt{g_{ii}} dy \quad \Longrightarrow \quad \sqrt{g_{ii}} = \frac{d}{dn_i} l_i(t, n_i) \,.$$

If we assume that $A_i^2 \sim const.$ and that our solution is $SO(3)$ symmetric, we obtain

$$g_{ij} = A(t)\delta_{ij} \,,$$

where $A(t)$ is a time-dependent amplitude. We obtain a homogeneous and isotropic metric, as expected.

The BFSS model is a non-perturbative formulation of M-Theory compactified to 10 dimensions. The (Euclidean) action of the BFSS model is given by

$$S_{BFSS} = \frac{1}{2g^2}\int_0^\beta dt \mathrm{Tr}\left(\frac{1}{2}(D_tX_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \theta^T D_t\theta - \theta^T\Gamma^i[\theta, X_i]\right)$$

where the covariant derivative is defined by $D_t = \partial_t - i[A(t), \cdot]$. Here, the X^i's are a set of 9 $N \times N$ hermitian matrices, the θ's are spinors of $SO(9)$, and $A(t)$ is an $N \times N$ matrix which we will take to be constant.

[T. Banks, W. Fischler, S. H. Shenker and L. Susskind, Phys. Rev. D **55**, 5112-5128 (1997)

[arXiv:hep-th/9610043 [hep-th]].]

Power spectrum of scalar and tensor perturbations

Let us assume the universe is described by a thermal state of the BFSS model. For a box of size L, the power spectra can be computed directly from the partition function of the system.

$$Z = \int dA d\psi e^{-S_{BFSS}}$$

For exemple, the dimensionless power spectrum $P_\psi(k)$ of scalar perturbations is sourced by energy density perturbations

$$P_\psi(k) = 16\pi^2 G^2 k^{-4} \delta\rho^2 \quad , \quad \delta\rho^2 = \frac{T^2}{L^6} C_V \, ,$$

which are proportional to the heat capacity of the system.

Similarly, the dimensionless power spectrum $P_h(k)$ of scalar perturbations is sourced by the transverse pressure perturbations of the system.

$$P_h(k) = 16\pi^2 G^2 k^{-4} C^{ij}_{ij} \quad , \quad C^{ij}_{ij} = \alpha \frac{T}{L^2} \frac{\partial \tilde{p}}{\partial L}$$

For a space-time described by the thermal BFSS model, the spectrum of scalar and tensor perturbations is given by

$$P_{\phi,h}(k) \sim (l_s m_{pl})^{-4}\, k^2 \langle R^2 \rangle \quad , \quad R^2 = \frac{1}{N} \mathrm{Tr}(X_0^{i\,2})\,.$$

Since $\langle R^2 \rangle \sim k^{-2}$, we obtain a scale-invariant power spectrum of amplitude $\mathcal{A} \sim (l_s m_{pl})^{-4}$ for both scalar and tensor perturbations!

It remains to show that $SO(10)$ is broken to $SO(4) \times SO(6)$ as the temperature of the system decreases. At high temperatures, the BFFS action can be approximated as

$$S_{BFSS} \approx -\frac{\beta}{4g^2}\text{Tr}[A^\mu, A^\nu]^2 + \frac{i\beta}{2g^2}\sum_r \psi_{-r}[A, \psi_r] + ...$$

In the IKKT model, it was shown that fermion lead to symmetry breaking. This was done by adding and substracting the Gaussian action

$$S_0 = \sum_{\mu=1}^{D} \frac{N}{v_\mu} Tr\left(A_\mu A_\mu\right) + \frac{N}{2}\sum_{a=1}^{N^2-1} \psi^a_\alpha M_{\alpha\beta}\psi^a_\beta\,,$$

to the IKKT action. Minimizing the free energy with respect to v_μ and $M_{\alpha\beta}$, we find that $SO(4) \times SO(6)$ is the prefered symmetry breaking breaking pattern!

Summary of the results

We explored a early universe scenario in which a 4 dimensional space is emergent in the IKKT and BFFS model.

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- IKKT model:
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  - We are not able to compute the spectrum of cosmological perturbations yet. However, we now have a proposal for the space-time metric.

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  - We are not able to compute the spectrum of cosmological perturbations yet. However, we now have a proposal for the space-time metric.
- BFSS model
  - The universe begins in a high temperature $SO(9)$ symmetric state. There are good reasons to think that the $SO(9)$ symmetric state is broken to $SO(3) \times SO(6)$ at lower temperatures (still under study).
  - The universe emerges with a scale invariant spectrum of scalar and tensor perturbations.

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Thank You

for your attention.